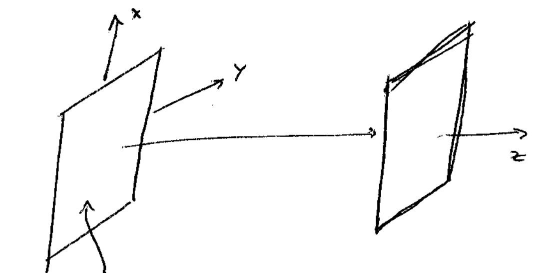


$E_x(x,y,z,t)$

$z=0$ $\quad$ z

$f(x,y)\cdot e^{j\omega t}$

$$E_x(x,y,z,t) = \frac{e^{j(\omega t - kz)}}{j\lambda z} \cdot \iint f(x',y') \cdot e^{-j\frac{\pi}{\lambda z}\left[(x-x')^2+(y-y')^2\right]} e^{-j2\pi(ux'+vy')} \, dx'dy'$$

$\downarrow$ F.T.

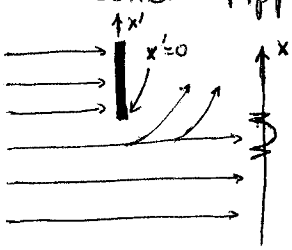
$$= \frac{e^{j(\omega t - kz)}}{j\lambda z} \cdot \iint F(u,v) \cdot e^{+j\pi(u^2+v^2)\lambda z} \cdot e^{+j\pi 2(ux'+vy')} \, du dv$$

Periodic $f(x,y)$'s

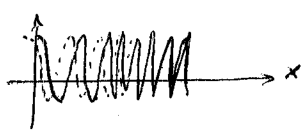
$$F(u,v) = \sum_n \sum_m F_{nm} \cdot \delta\left(u - \frac{n}{\alpha}\right) \cdot \delta\left(v - \frac{m}{\beta}\right)$$

Examples

- Fresnel ripples



- where there is no boundary, expect uniform illumination.
- at the boundary:

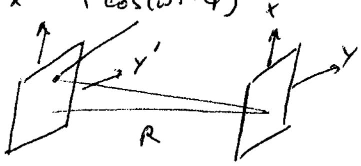


- that is the +,-ive parts of this curve when integrated over all light that gets through does not result in uniform illumination.

Complex notation

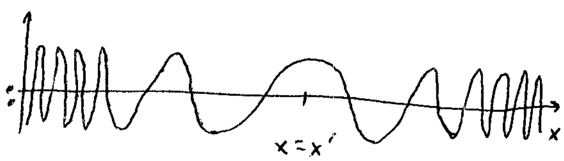
Real
$$E_x(x,y,z,t) = \frac{1}{\lambda z} \cdot \iint f(x,y) \cdot \underbrace{\cos\left(\omega t - kz - \frac{\pi}{\lambda z}\left[(x-x')^2+(y-y')^2\right] - \frac{\pi}{4}\right)}_{\cos(\phi)} dx'dy'$$

x' $\quad f\cos(\omega t+\phi)$ $\quad x$



$f(x',y')$ $\qquad f'\cos(\omega t+\phi')$

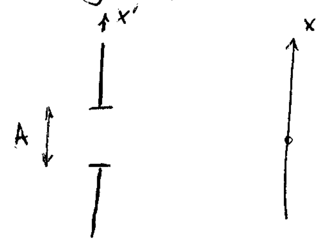
@ $t=t_1$, $z=z_1$, $y=0$:



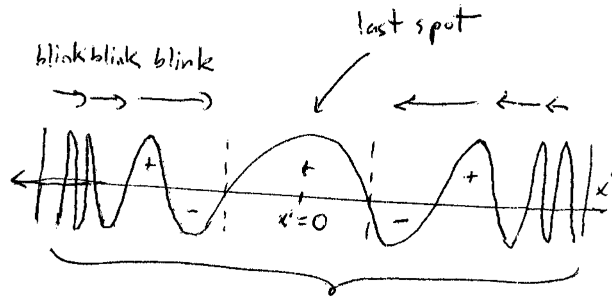
$\cos\left[\frac{\pi}{\lambda z}(x-x')^2 + \phi\right]$

Examples

- Blinking Spot

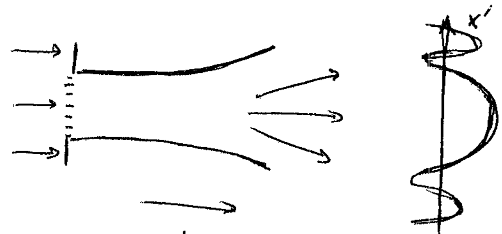


$$\int_{-\frac{A}{2}}^{\frac{A}{2}} \cos\left[\frac{\pi(x-x')^2}{\lambda z}\right] dx'$$



– spot blinks because as you go farther away (large z), you are integrating over a smaller x'.

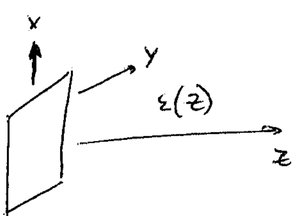
- Pattern Recognition
- Talbot Images
- Square Aperture
- Square Aperture with Grating
- Circular Aperture
- Grating in 2-Dimensions
- Triangle



$\underline{\underline{\varepsilon(z)}}$, $\mu = \text{const.}$, $\sigma = 0$, $\rho = 0$

$$\nabla \cdot \vec{D} = \rho = 0 \Rightarrow \nabla \cdot (\varepsilon \vec{E}) = 0$$

$$\left.\begin{aligned} \nabla \times \vec{H} &= \varepsilon \cdot \frac{\partial \vec{E}}{\partial t} \\ \nabla \times \vec{E} &= -\mu \cdot \frac{\partial \vec{H}}{\partial t} \end{aligned}\right\} \Rightarrow \nabla \times \nabla \times \vec{E} = -\nabla^2 \vec{E} + \nabla(\nabla \cdot \vec{E}) = -\mu \cdot \frac{\partial}{\partial t}(\nabla \times \vec{H}) = -\mu\varepsilon \cdot \frac{\partial^2 \vec{E}}{\partial t^2}$$



E_x

assume $\frac{\partial E_x}{\partial x}, \frac{\partial E_y}{\partial y} = 0$

$$\cancel{\frac{\partial E_x}{\partial x}}^{0} + \cancel{\frac{\partial E_y}{\partial y}}^{0} + \frac{\partial E_z}{\partial z}^{\,0} = 0$$

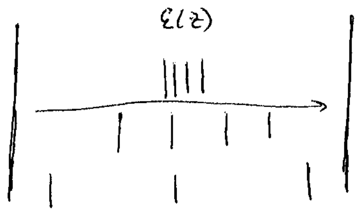
$$\cancel{\frac{\partial^2 E_x}{\partial x^2}}^{0} + \cancel{\frac{\partial^2 E_x}{\partial y^2}}^{0} + \frac{\partial^2 E_x}{\partial z^2} = +\mu\varepsilon \frac{\partial^2 E_x}{\partial t^2}$$

$$\frac{\partial^2 E_x}{\partial z^2} = -\omega^2 \mu \varepsilon(z) \cdot E_x$$

Assume $E_x(z,t) = A \cdot e^{j\omega t} \cdot \boxed{\underbrace{e^{-jkz}}_{k = \frac{\omega}{v} = \omega\sqrt{\mu\varepsilon}}}$ tempted to do this but since $k(z)$, it doesn't work.

- works for slow-varying $\varepsilon(z)$ (compared to wavelength λ)

$$\varepsilon(z) = \int \tilde{\varepsilon}(u) \cdot e^{-j2\pi uz} \cdot du$$



$\Rightarrow$ multiple propagation constants $k^{(1)}, k^{(2)}, \ldots$

<u>Rest of class</u>

Reconcile differences across the boundary

ε_1	ε_2
σ_1	σ_2
ρ_1	ρ_2

- Interfaces
  - some reflection, transmission
  - absorption, propagation
  - waveguides, antennas