EE 151
Electromagnetic Engineering

Caltech
Fall '05

Homework 4

Due Tuesday, Oct. 25^{th} in class

1. Heinrich Hertz was the first person to produce the transmission and reception of electromagnetic waves. In one of his classic experiments (**Fig.** 1), he used an LC oscillator to generate high frequency oscillating sparks in a tiny air gap between two metal wires. The sparks generated electromagnetic waves. He then used a metal ring with a spark air gap as the detector. When he put a mirror at a certain distance from the oscillator to reflect the electromagnetic wave, he found the strength of the sparks in the metal ring changed with distance from the oscillator. At some positions the sparking disappeared.

    a). What is the quantity measured by the sparking intensity? Explain why the sparking strength changed with position.

    b). The three positions Hertz measured when the sparking disappeared were 0.1 meters, 2.9 meters and 5.7 meters from the LC oscillator. The oscillating frequency of the LC circuit was 0.14GHz. Based on these, Hertz calculated the propagation speed of the generated electromagnetic wave. Repeat this calculation.

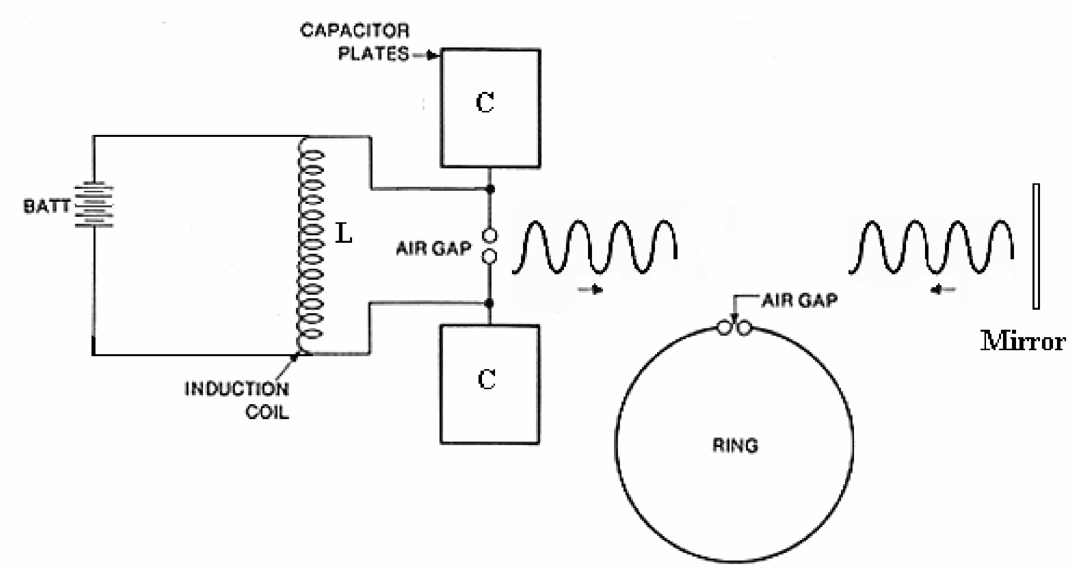

Figure 1

2. Consider an input wave with the distribution of the electric field given by

$$E(x, z = 0) = 0.5\,[1 + \cos(2\pi u_0 x)] \times \mathrm{rect}(x/A)e^{j\omega t}$$

At some distance z, the diffraction of the above function can be approximated by 3 non-overlapping rectangles. Determine the minimum distance z when the rectangles are no longer overlapping.

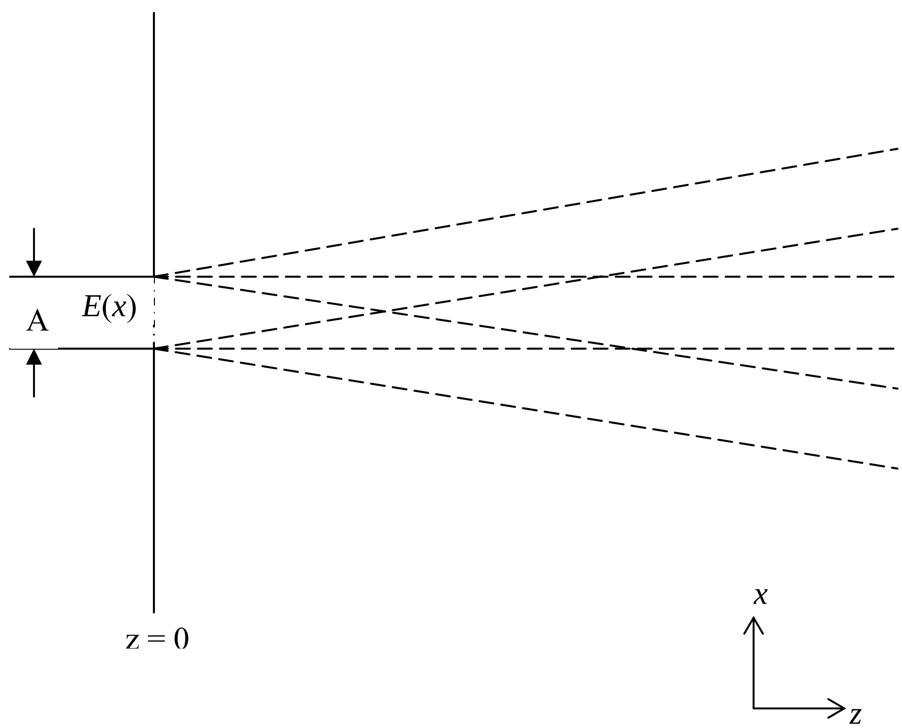

Figure 2

3. Consider a one-dimensional periodic object with period Λ and an amplitude transmittance having an arbitrary periodic profile. Show that, if $\Lambda >> \lambda$, at certain distances behind this object, perfect images of the amplitude transmittance are found. At what distances do these "self-images" appear.

4. An atom can be regarded as a fixed nucleus (charge $+e$) surrounded by an electron cloud of charge $-e$ and mass m which oscillates back and forth in response to an applied time-varying electric field. The electron cloud can be considered as a uniform charge distribution in a sphere of radius R. Assume the applied E field is along x direction and its frequency is ω. The electron cloud experiences three forces: the force due to the applied field, the restoring force due to the coulomb attraction of the nucleus, and a frictional force which results in damping of the oscillation. This damping force is proportional to the mass and velocity of the electron cloud and can be expressed as:

$$F_d = -\gamma m\dot{x} \quad \text{where } \gamma \text{ is a positive constant.}$$

Based on this model, calculate the complex permittivity and skin depth of a dielectric material given the density of atoms in the material is N.

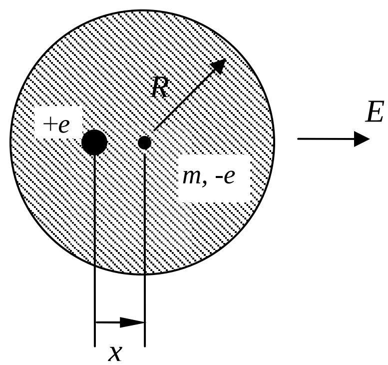

Figure 4