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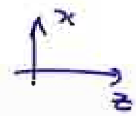
plane wave : $e^{j\omega t} e^{-jkz}$

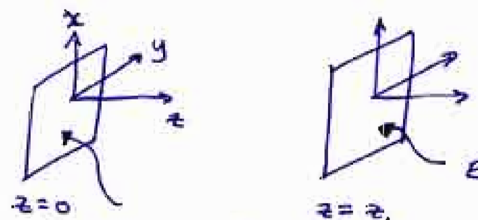
laser - Gaussian : $e^{-y^2/\alpha} e^{-x^2/\alpha} e^{j\omega t} e^{-jkz}$



$\Delta\theta = \frac{\lambda}{W}$

Diffraction demo : grating - different periods

Diffraction



$\left.\begin{array}{l} x' = x \\ y' = y \end{array}\right\} @ z = z_1$

$E(x,y,z=z_1) = e^{j\omega t} \cdot g(x',y') \cdot e^{-jk_z z_1}$

$E(x,y,z=0) = e^{j\omega t} \cdot f(x,y)$

given $f \longrightarrow$ find g

"Example" : plane wave - $f(x,y,z) = \underbrace{e^{-jk_x x} \cdot e^{-jk_y y}}_{\text{input pattern}} \cdot \underbrace{e^{-jk_z z}}_{=1}$ $@ z=0$ ($z = 0$)

after propagation of z_1, input pattern doesn't change

$\Rightarrow \quad g(x',y',z) = \underbrace{e^{-jk_x x'} \cdot e^{-jk_y y'}} \cdot \underbrace{e^{-jk_z z_1}}_{\text{phase delay depends on } k_x, k_y \text{ since}}$ $@ z = z_1$

$k_x^2 + k_y^2 + k_z^2 = \omega^2 \mu \epsilon$

$v = \lambda \nu \Rightarrow \frac{2\pi}{\lambda} = k$

Fourier Transform

$$s(t) = \int S(\omega) \cdot e^{-j\omega t} \cdot \frac{d\omega}{2\pi}$$

Example

$$s(t) = \cos(\omega_0 t) \overset{\mathcal{F}}{\Rightarrow} S(\omega) = \delta(\omega-\omega_0) + \delta(\omega+\omega_0)$$

Back to diffraction....

$$f(x,y) = \iint F(k_x,k_y) \cdot e^{-jk_x x} \cdot e^{-jk_y y} \cdot \frac{dk_x dk_y}{(2\pi)^2} \qquad @\ z=0$$

$$g(x',y') = \iint \underbrace{F(k_x,k_y)}_{F=|F|\cdot e^{j\phi}} \cdot \underbrace{e^{-jk_x x'} \cdot e^{-jk_y y'} \cdot e^{-jk_z z_1}}_{} \cdot \frac{dk_x dk_y}{(2\pi)^2} \qquad @\ z=z_1$$

$$k_x^2 + k_y^2 + k_z^2 = \omega^2 \mu \varepsilon = \left(\frac{2\pi}{\lambda}\right)^2 = k^2$$

$$\Rightarrow k_z = \sqrt{k^2 - k_x^2 - k_y^2} = k\sqrt{1 - \frac{k_x^2+k_y^2}{k^2}}$$

Remember convolution....

$$s(t) * h(t) = \int s(t')\, h(t-t')\, dt' \overset{\mathcal{F}}{\Rightarrow} S(\omega)\, H(\omega)$$

Going back....

Let us approximate for k_z using Taylor's approximation:

$$k_z \approx k - \frac{k_x^2 + k_y^2}{2k} \qquad \text{since} \quad \sqrt{1+\alpha} \approx 1 - \frac{\alpha}{2}$$

This is known as the <u>paraxial approximation</u>.

Remember FT's ...

$$e^{-\pi k^2} \overset{\mathcal{F}}{\Longrightarrow} e^{-\pi v^2}$$

$$e^{-j\pi k^2} \overset{\mathcal{F}}{\Longrightarrow} e^{+j\pi v^2}$$

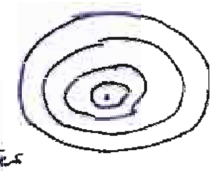
Fresnel diffraction equation

$$g(x',y') \equiv \frac{e^{-jkz}}{j\lambda z} \iint f(x',y') \cdot e^{j\frac{\pi}{\lambda z}\left[(x-x')^2+(y-y')^2\right]} dx'dy'$$

Spherical wave

$$E_{sph} = \frac{A}{j\lambda r} \cdot e^{j(\omega t - \vec{k}\cdot\vec{r})}$$

spherical coordinates



assuming it's centered at $r=0$

$$r = \sqrt{(x-x')^2+(y-y')^2+ z_1^2} \simeq z_1 + \frac{(x-x_1)^2+(y-y_1)^2}{2z_1}$$

using the paraxial approximation

Assume $(x-x_1)^2, (y-y_1)^2 << z_1$ s.t. we can approximate $r \simeq z_1$.

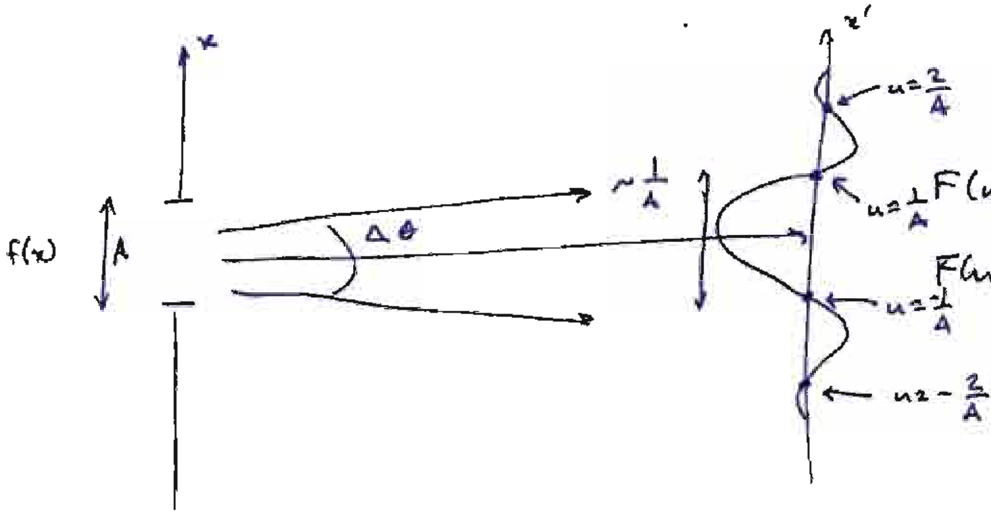
Far field approximation (since $z_1 \to \infty$)

In 1-D: $e^{j\frac{\pi}{\lambda z_1}(x^2+x'^2-2xx')} \simeq e^{j\frac{\pi}{\lambda z_1}x'^2} e^{-j\frac{\pi}{\lambda z_1}2xx'}$ ignore x^2 term since $x^2 << z_1$, $\frac{x^2}{z_1} \approx 0$.

$$g(x') = \frac{e^{jkz_1}}{j\lambda z_1} \cdot e^{j\frac{\pi}{\lambda z_1}x'^2} \cdot \int f(x) e^{-j\frac{2\pi}{\lambda z_1}xx'} dx = \frac{e^{j()}}{j\lambda z_1} \cdot F(u)$$

$\nwarrow$ F.T. of $f(x)$

$$\frac{x'}{\lambda z_1} = u \longleftarrow \text{frequency}$$



$$f(x) = \mathrm{rect}\left(\frac{x}{A}\right)$$

$$\downarrow \mathcal{F}$$

$$F(u) = A\,\mathrm{sinc}(Au)$$

$$F(u) = \frac{A \sin \pi A u}{\pi A u}$$

$$\Delta u \cong \frac{1}{A} \quad , \quad u = \frac{x'}{\lambda z_1} \Rightarrow \Delta x' = \frac{\lambda z_1}{A}$$