Surface Plasmon-Polaritons

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So far you have learned about many types of waveguides. Some have metal walls that can be approximated as perfect conductors, so that waves can be confined inside waveguides by reflecting back and forth between the walls almost perfectly. Some have a core with a high refractive index and a cladding with a lower refractive index, so waves are confined via total internal reflection. Here I am going to talk about a very funny kind of waveguide mode, called Surface Plasmon-Polariton (SPP). Despite its intimidating name, which I will explain towards the end, SPP can be understood using very basic electromagnetics.

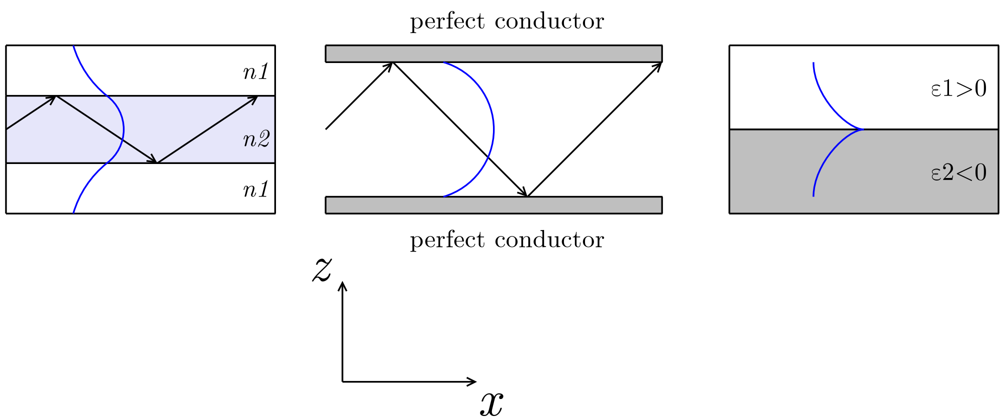

FIG. 1: Dielectric slab waveguide, metal waveguide, and a surface plasmon polariton waveguide. The blue curve is a sketch of the waveguide mode profile for each waveguide.

An SPP can be supported simply by putting a dielectric with $\epsilon > 0$ together with a material with $\epsilon < 0$. It turns out that at optical frequencies, certain metals have a negative real part of ϵ, and a relatively small imaginary part, so loss is actually resonably low. For example, at the wavelength 1550 nm, silver has an $\epsilon = (-120 + j11)\epsilon_0$. To simplify the analysis, I will assume the ϵ_2, the permittivity of the metal, is real.

The reason I say an SPP is funny is because it lives on the *surface* between the dielectric and the metal. Unlike other types of waveguides, there is *no* propagating waves bouncing back and forth, and on both sides of the interface both of the waves are actually *evanescent*. The discontinuity of the mode profile on the interface is supported by the opposite signs of the permittivity on both sides.

To derive the SPP mode, consider the TM polarization, where the magnetic field **H** points in the y direction (into the board). The SPP mode can be derived by matching boundary conditions, just like deriving reflection and transmission coefficients.

Boundary condition tells us that H_y is the same on both sides. In the dielectric, I'll let the fields to be

$$\mathbf{E}(z>0) = (E_x\hat{\mathbf{x}} + E_z\hat{\mathbf{z}})\exp(jk_x x + jk_z z), \tag{1}$$

$$\mathbf{H}(z>0) = H_y\hat{\mathbf{y}}\exp(jk_x x + jk_z z), \tag{2}$$

and in the metal,

$$\mathbf{E}(z<0) = (E_x\hat{\mathbf{x}} + E'_z\hat{\mathbf{z}})\exp(jk_x x + jk'_z z), \tag{3}$$

$$\mathbf{H}(z<0) = H_y\hat{\mathbf{y}}\exp(jk_x x + jk'_z z). \tag{4}$$

Remember that the tangential electric field and tangential magnetic field, E_x and H_y, are the same on both sides. Same for the tangential wave vector, k_x. Obviously,

$$k_x^2 + k_z^2 = \omega^2\mu_0\epsilon_1, \tag{5}$$

$$k_x^2 + k_z'^2 = \omega^2\mu_0\epsilon_2. \tag{6}$$

The normal displacement field is continuous, so I have

$$\epsilon_1 E_z = \epsilon_2 E'_z, \quad E'_z = \frac{\epsilon_1}{\epsilon_2}E_z. \tag{7}$$

Using Gauss's Law in the dielectric,

$$\nabla\cdot\mathbf{D} = 0, \tag{8}$$

$$E_x\frac{\partial}{\partial x}\exp(jk_x x + jk_z z) + E_z\frac{\partial}{\partial z}\exp(jk_x x + jk_z z) = 0, \tag{9}$$

$$jk_x E_x + jk_z E_z = 0, \tag{10}$$

$$E_z = -\frac{k_x}{k_z}E_x. \tag{11}$$

Similarly, inside the metal,

$$E'_z = \frac{\epsilon_1}{\epsilon_2}E_z = -\frac{k_x}{k'_z}E_x. \tag{12}$$

Combining Eqs. (11) and (12), we have

$$\frac{\epsilon_1}{\epsilon_2}\frac{k_x}{k_z}E_x = \frac{k_x}{k'_z}E_x \tag{13}$$

$$\frac{k_z}{k'_z} = \frac{\epsilon_1}{\epsilon_2}. \tag{14}$$

We want the SPP mode to exponentially decay away from the interface, i.e. k_z to be imaginary. If we want the modes to decay away from the interface in both sides, k_z and k'_z must both be imaginary as well as have opposite signs. This only happens if ϵ_1 and ϵ_2 have opposite signs. But this is exactly what we've assumed at the very beginning! So all is good.

Eq. (14) tells you that there is a restriction on the ratio between the (imaginary) normal wave vectors on both sides. Since k_z and k'_z are both related to k_x by the dispersion relations Eqs. (5) and (6), this means that there is also a restriction on k_x. Taking the square of Eq. (14), we have

$$\frac{k_z^2}{k_z'^2} = \frac{\omega^2\mu_0\epsilon_1 - k_x^2}{\omega^2\mu_0\epsilon_2 - k_x^2} = \frac{\epsilon_1^2}{\epsilon_2^2} \tag{15}$$

$$k_x^2 = \frac{\omega^2\mu_0}{1/\epsilon_1 + 1/\epsilon_2}. \tag{16}$$

So an SPP only exists for one specific k_x. Recall that ϵ_1 and ϵ_2 have opposite signs. For k_x to be real and the SPP to be propagating, k_x^2 must be positive, so

$$\frac{1}{\epsilon_1} > -\frac{1}{\epsilon_2} = \frac{1}{|\epsilon_2|}, \tag{17}$$

$$|\epsilon_2| > \epsilon_1, \tag{18}$$

and the magnitude of the metal permittivity must be larger than the magnitude of the dielectric permittivity for an SPP to exist.

The most useful aspect of an SPP is the fact that its k_x can be very large:

$$k_x^2 = \frac{\omega^2\mu_0}{1/|\epsilon_1| - 1/|\epsilon_2|} \tag{19}$$

$$= \frac{\omega^2\mu_0\epsilon_0}{1/|\epsilon_{R1}| - 1/|\epsilon_{R2}|} = \frac{k_0^2}{1/|\epsilon_{R1}| - 1/|\epsilon_{R2}|} \tag{20}$$

It is proportional to the difference between $1/|\epsilon_{R1}|$ and $1/|\epsilon_{R2}|$, so if $|\epsilon_{R1}|$ and $|\epsilon_{R2}|$ are very close to each other, k_x can be much larger than the free space wave vector.

To see why having a large k_x is useful, imagine that we have an image with a transverse profile $A(x)$. The spatial frequency amplitude is

$$a(k_x) = \int dx A(x) \exp(-jk_x x). \tag{21}$$

The higher spatial frequencies contain the finer details of the image. Each k_x roughly corresponds to a feature size of $\Lambda \sim 2\pi/k_x$. These spatial frequency components propagate in z according to the dispersion relation

$$a(k_x, z) = a(k_x) \exp\left(j\sqrt{k_0^2 - k_x^2} z\right) \tag{22}$$

$$A(x, z) = \int \frac{dk_x}{2\pi} a(k_x) \exp\left(j\sqrt{k_0^2 - k_x^2} z\right) \tag{23}$$

The problem is that for the very fine details of the image, k_x can be larger than k_0, or the feature size can be smaller than λ_0. then $\sqrt{k_0^2 - k_x^2}$ becomes imaginary, and the fine details of the image would become *evanescent*, and decay exponentially away from the image. Any conventional optical system is unable to pick up the evanescent wave, and the high resolution information of the image is lost. However, if we place a metal slab very close to the image, the evanescent wave can be coupled to the SPP on one side of the slab, and then to the other side of the slab, so that the evanescent wave can be transmitted for a longer distance.

The reason why this is called a Surface Plasmon-Polariton is, firstly, because it lives on the *surface* of a dielectric-metal interface. Secondly, the negative permittivity of a metal is due to the oscillations of free electrons, which can be understood as a *plasma* effect. Thirdly, a *polariton* is defined as an EM wave strongly coupled to the excitation of a material. In this case, the EM wave is strongly coupled to the oscillations of the free electrons in the metal, so it is called plasmon-polariton.