

### Homework 3

Due Tuesday, April 19<sup>th</sup> in class

1. An ideal parallel-plate capacitor is shown in **Fig. 1**. The radius of each plate is  $a$ . The spacing between the two plates is  $h$ . A time-varying voltage  $V(t) = V_0 \sin(\omega t)$  is applied across the capacitor. Assume uniform field inside the capacitor and ignore fringing effects.

(a) A rectangular-shaped open metal loop is inserted in the capacitor. The surface of the loop is parallel to the axis of the capacitor. The dimensions of the loop are width  $w$  and length  $l$ . The insertion depth is  $b$ . Calculate the induced voltage  $V_{\text{ind}}$  across the loop.

(b) Show that

$$\frac{dU}{dt} = \oint_s \mathbf{E} \times \mathbf{H} \cdot d\mathbf{s},$$

Where  $U(t)$  is the total electromagnetic energy stored inside the capacitor,  $\mathbf{E}$  and  $\mathbf{H}$  are, respectively, the electric and magnetic field vector. The integral is evaluated over the surface that encloses the volume between the two circular plates.

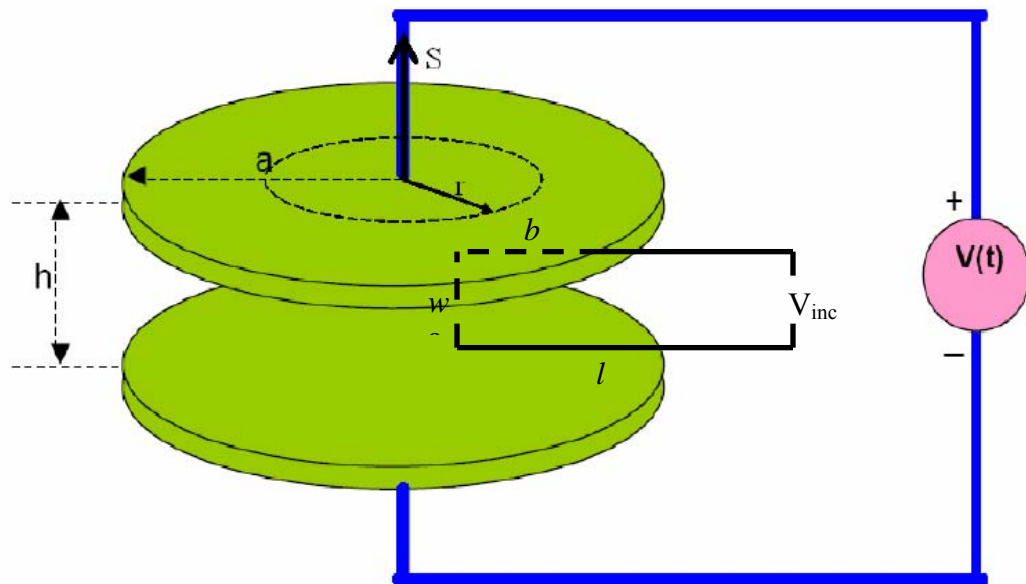


Figure 1

2. The electric field component of an electromagnetic wave in free space is given by

$$e(y, z, t) = i_x E_0 \cos(ay) \cos(\omega t - bz)$$

where  $i_x$  denotes a unit vector along x direction.

- Find the corresponding magnetic field.
- Find the relationship between the constants a, b, and  $\omega$  such that all of Maxwell's equations are satisfied.
- Assuming that this wave can be regarded as a sum of two uniform plane waves, determine the direction of propagation of the two component waves.

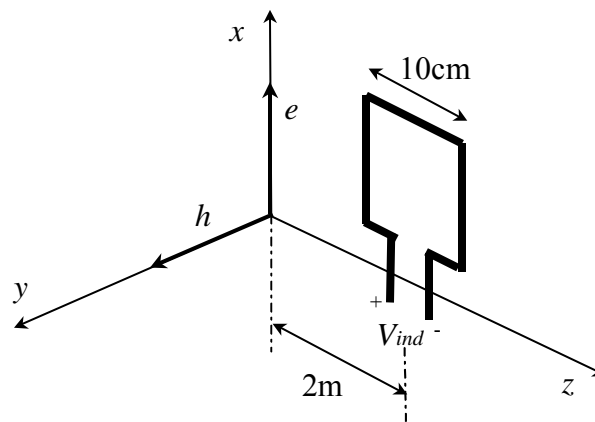
3. A uniform plane electromagnetic wave propagates in free space with electric and magnetic field components as shown in **Fig. 2**.

$$e = i_x E_0 \cos(\omega t - \beta z)$$

$$h = i_y \frac{E_0}{\eta} \cos(\omega t - \beta z)$$

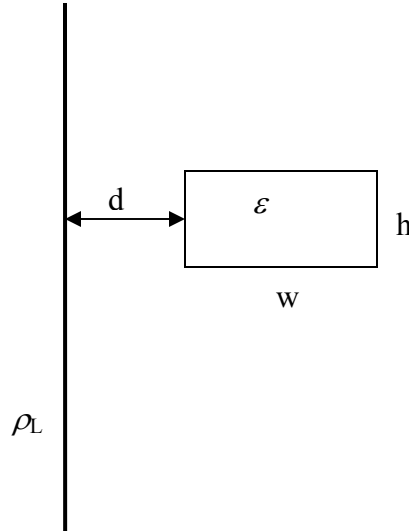
The wave frequency is 300 MHz and the electric field amplitude  $E_0=1\text{V/m}$ . A square loop with side length  $a=10\text{cm}$  is placed at  $z=2\text{m}$  as shown in the figure.

- Find the voltage  $V_{ind}$  induced at the terminals of the loop.
- Repeat (a) if the loop is located at a distance  $d=3\text{m}$  from the x axis instead of 2m as shown, and compare the answers from (a) and (b)
- Repeat a) for  $d=2\text{m}$  if the loop is rotated for  $90^\circ$ , i.e. if its normal is now parallel to the z axis.



**Figure 2**

4. A rectangular dielectric plate (dielectric constant  $\epsilon$ , width  $w$ , height  $h$ , and thickness  $t$ ) is located at a distance  $d$  from an infinite long uniform line charge with charge density  $\rho_L$ . Assume  $t \ll d$ . Calculate the force acting on the plate.



**Figure 3**

5. (FEMLAB) At very high electric fields, the refractive index of a material becomes intensity-dependent,

$$n = n_0 + n_2 I$$

Optical waves propagating in such a medium show some interesting behaviors. Use FEMLAB Electromagnetic Module (2D) to simulate a Gaussian beam propagating in such a material. Assume the free space wavelength is  $\lambda_0 = 1\mu\text{m}$ , the Gaussian beam intensity profile is  $I(r) = I_0 \exp(-r^2/a^2)$ , where  $I_0 = 10\text{GW}/\text{cm}^2$  and  $a = 1\mu\text{m}$ . Show the intensity plot over a propagation distance of  $10\mu\text{m}$  for  $n_2 I_0 = 10^{-3}$ ,  $0.1$  and  $0.5$ . Explain the simulation results.