

Homework 4

Due Tuesday, April 26th in class

1. Region 1, $z < 0$, and region 2, $z > 0$, are described by the following parameters: $\epsilon'_1 = 100$ pF/m, $\mu_1 = 25$ μ H/m, $\epsilon''_1 = 0$, $\epsilon'_2 = 200$ pF/m, $\mu_2 = 50$ μ H/m, and $\epsilon''_2/\epsilon'_2 = 0.5$. If $\mathbf{E}_1^+ = 600 e^{-\alpha_1 z} \cos(5 \times 10^{10} t - \beta_1 z) \mathbf{a}_x$ V/m, find:
 - (a) α_1
 - (b) β_1
 - (c) \mathbf{E}_{s1}^+
 - (d) \mathbf{E}_{s1}^-
 - (e) \mathbf{E}_{s2}^+
2. In a dispersive material the refractive index n depends on the frequency. Electromagnetic pulses propagating in such materials will experience broadening. Consider a plane wave of frequency ω_0 that is temporally modulated by a Gaussian pulse $E(z=0, t) = E_0 e^{-\frac{1}{2}(\frac{t}{T})^2}$.
 - a). Show that the pulse envelope propagates approximately at the velocity $v_g = 1/(d\beta/d\omega)|_{\omega_0}$, where β is the propagation constant.
 - b). If n varies linearly with frequency $n(\omega) = n_0 + \Delta n(\omega - \omega_0)$, determine when the pulse will spread to twice its initial width.
3. Consider a one-dimensional periodic object with period Λ and an amplitude transmittance having an arbitrary periodic profile. Show that, if $\Lambda \gg \lambda$, at certain distances behind this object, perfect images of the amplitude transmittance are found. At what distances do these "self-images" appear.